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(10) D. H. Rock.

(12) 15p.

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[illegible] ... to infinity. The ... theory ... by Riabouchinski, artificially ... which closes the cavity ... lutions to the problem of flow ... are presented for both models, and calculations ... drag coefficient ... numbers, N, between 0 and 1.3. Both ... found to ... essentially the same results over this entire range. The drag coefficient grows almost linearly with ... given to a close approximation by the formula ... where $C_{Do} = \frac{2\pi}{4+\pi}$ is the classical result of ... drag on a flat plate with infinite cavity.

Foreword: The data and conclusions presented here are for the use of the personnel of the Naval Ordnance Laboratory. They may not represent the final judgment of the Laboratory.

Refs: (a) Milne-Thompson: "Theoretical Hydrodynamics", Macmillan, Chapters 11, 12.

(b) Kreisel, G.: "Cavitation with Finite Cavitation Numbers", Admiralty Research Laboratory Report.

(c) Riabouchinski, D: "Sur les singularités des mouvements fluides", Proceedings of the Second International Congress for Applied Mechanics, pp. 512-518.

Encl: (A) (HW) Figures 1 through 4.
(B) (HW) Tables 1 and II.

[illegible] It is well known that ... flow of a liquid ... [illegible] ... experiments, ... [illegible] ... vapor.

[illegible] ... steady ... depend ... on the static pressure P in the undisturbed stream, the cavity pressure p (equal to the vapor pressure in a water tunnel), and the dynamic pressure $\frac{1}{2}\rho V^2$, where ρ = density of the liquid, V = free stream velocity. For a given obstacle the extent of cavitation is determined by the cavitation number,

$$K = \frac{P - p}{\frac{1}{2}\rho V^2}$$

The size of the cavity increases as K approaches zero, that is, as P approaches p or as $P/\frac{1}{2}\rho V^2$ approaches zero. In water tunnel experiments, where there are limitations on the maximum attainable velocity V, the cavitation number is reduced by letting P approach p; in water entry experiments the dynamic head is controllable, so that $P/\frac{1}{2}\rho V^2$ can be made as small as desired.

3. The theory of the cavity formed when the static pressure in the liquid is equal to the pressure in the cavity is identical in the two-dimensional case with the theory of the infinite wake (Figure 1-a) which was first explored by Kirchhoff and Helmholtz, and later extended by Levi-Civita and others; see reference (a) for some details of this work. Physically the infinite cavity cannot occur, but can be considered as the limiting case of that occurs as the dynamic head increases without limit relative to the static pressure in the undisturbed liquid, or as the static pressure approaches the cavity pressure.

4. The Kirchhoff flow is limited to infinite cavities and zero cavitation number. An exact theory of the finite cavity corresponding to non-zero cavitation number has only recently been given. This theory is attributed to H. Wagner, although unpublished by him, and was developed independently in 1945 by G. Kreisel of the Admiralty Research Laboratory, England (reference b). The essential feature of the theory is the occurrence of a reverse jet extending from the rear of the cavity "through" the obstacle to infinity, as shown in Figure 1-b. Although this seems physically objectionable, it is a good description of what has been observed experimentally at the Naval Ordnance Laboratory. Jets do indeed occur in finite cavities, although they are either weakened by the turbulent mixing at the rear of the cavity, or finally collide with the obstacle.

[illegible] ... in that it ... satisfying the ... [illegible]

[illegible] ... [illegible]

[illegible] ... two-dimensional parallel flow ... [illegible] ... sure P against a flat plate**. We shall assume that the free (as yet unspecified) streamlines separate from the obstacle at its extremities ... these streamlines enclose a region of constant pressure p, ($p \leq P$). The problem is to determine the flow completely, including the shape of the free bounding streamlines. The condition on the free boundaries is given by Bernoulli's equation

$$P + \tfrac{1}{2}\rho V^2 = p + \tfrac{1}{2}\rho U^2,$$

where U is the (constant) flow velocity on the free streamlines. Thus

$$\frac{U^2}{V^2} = \frac{P - p}{\tfrac{1}{2}\rho V^2} + 1 = 1 + N.$$

When $P = p$, and therefore $N = 0$, the classical Kirchhoff type flow shown in Figure 1-a prevails.

8. For the case $P > p$, consider the flow shown in Figure 1-b. We shall need only the upper half of the flow plane, which will be designated the z-plane. The flow problem is solved once the complex velocity potential, $w(z) = \phi + i\psi$, is determined, where ϕ = velocity potential, ψ = stream function.

* This is required by the fact that the pressure always increases from the cavity into the flow.

** The following treatment can very easily be applied to wedges also.

[illegible]

[illegible]

[illegible]

The mappings on the ζ-plane are given by the Schwarz-Christoffel transformations:

(1) Q-plane on the ζ-plane

$$\frac{dQ}{d\zeta} = \frac{K_1}{(\zeta - c)[(\zeta-1)(\zeta-b)]^{1/2}}$$

with subsidiary conditions as obtained from the known jumps in Q at 0 and C,

$$K_1 = c\sqrt{b}/2 = c\sqrt{c-1}\sqrt{c-b}$$

(A) $$\therefore \sqrt{c-1}\sqrt{c-b} = \sqrt{b}/2$$

(2) w-plane on the ζ-plane

$$\frac{dw}{d\zeta} = K_2 \frac{\zeta - c}{\zeta - b}$$

with subsidiary condition

$$K_2 = \psi_0/\pi(c-b) \quad .$$

From (1) it follows that

$$Q = \frac{1}{2}\log\frac{\sqrt{\zeta-b}+\sqrt{b}\sqrt{\zeta-1}}{\sqrt{\zeta-b}-\sqrt{b}\sqrt{\zeta-1}} + \log\frac{\sqrt{c-1}\sqrt{\zeta-b}+\sqrt{c-b}\sqrt{\zeta-1}}{\sqrt{c-1}\sqrt{\zeta-b}-\sqrt{c-b}\sqrt{\zeta-1}} + \log K$$

[illegible] therefore K_1 [illegible] and

[illegible] $\left(\frac{[illegible]}{[illegible]}\right)^{1/2}$ [illegible]

[illegible]

[illegible] number. From (2) it follows that

$$[illegible] = \frac{\psi_0}{\pi(c-b)} [illegible] - (c-b)\log(1 - [illegible]).$$

[illegible] length ℓ of the plate [illegible]

$$[illegible] (dz/d\zeta) [illegible] = \int [illegible] (dz/d\zeta)\,d\zeta$$

and

$$[illegible] = \frac{\psi_0}{\pi(c-b)U}\int_0^{\zeta}\left(\frac{\sqrt{\zeta-b}+\sqrt{b}\sqrt{\zeta-1}}{\sqrt{\zeta-b}-\sqrt{b}\sqrt{\zeta-1}}\right)^{1/2}\left(\frac{\sqrt{c-1}\sqrt{\zeta-b}+\sqrt{c-b}\sqrt{\zeta-1}}{\sqrt{c-1}\sqrt{\zeta-b}-\sqrt{c-b}\sqrt{\zeta-1}}\right)\frac{\zeta-c}{\zeta-b}\,d\zeta$$

The thickness of the jet is ψ_0/U. The shape of the cavity is given parametrically as a function of ζ ($0 \leq \zeta \leq b$) by the expression

$$\frac{1}{\pi}\frac{\psi_0}{(c-b)\,U}\int_0^{\zeta}\left(\frac{\sqrt{\zeta-b}+\sqrt{b}\sqrt{\zeta-1}}{\sqrt{\zeta-b}-\sqrt{b}\sqrt{\zeta-1}}\right)^{1/2}\left(\frac{\sqrt{c-1}\sqrt{\zeta-b}+\sqrt{c-b}\sqrt{\zeta-1}}{\sqrt{c-1}\sqrt{\zeta-b}-\sqrt{c-b}\sqrt{\zeta-1}}\right)\frac{\zeta-c}{\zeta-b}\,d\zeta$$

10. The drag is computed by integrating the excess pressure over the front face of the plate, neglecting the effect of the jet on the back face, however:

$$\text{Drag} = C_D\,\tfrac{1}{2}\rho\ell V^2 = \rho\int_0^{\ell/2}(U^2 - q^2)\,dy, \quad (C_D = \text{drag coefficient})$$

$$C_D = \frac{U^2}{V^2}\left(1 - \frac{\int_0^{\ell/2} q^2\,dy}{\frac{1}{2}\rho U^2}\right).$$

Since

$$\int_0^{\ell/2} q^2\,dy = -i\int_0^{i\ell/2}\left|\frac{dw}{dz}\right|^2 dz = -i\int_0^1 \frac{\overline{dw}}{dz}\frac{dw}{d\zeta}\,d\zeta = i\int_0^1 \frac{dw}{dz}\frac{dw}{d\zeta}\,d\zeta$$

then

$$C_D = \frac{U^2}{V^2}(1 - I_1/I_2) = (1 + K)\,(1 - I_1/I_2)$$

where

[illegible]

where [illegible] is the integrand [illegible] of integration in the ζ-plane excluding the singularities [illegible]. However, the path of integration is taken as the segment [illegible] of the real axis in the ζ-plane (corresponding to the path [illegible] in the flow plane), [illegible] the Cauchy principal value as the value of the integral, the real part of the result is the distance between stagnation points, while the imaginary part is the thickness of the jet which corresponds to the jump at [illegible].*

11. The integrals in the expression for C_D can be written in terms of elementary functions. C_D was evaluated for cavitation numbers between 0 and 1.5. The curve of drag coefficient vs. cavitation number is shown in Figure 2, and is seen to be almost a straight line. The variation of I_1/I_2 with cavitation number is shown in Figure 3; over the region considered the total variation remains less than 8%. To a good approximation, it follows

$$C_D = C_{D_0}(1+N) = 0.88(1+N)$$

where $C_{D_0} = 0.88$ is the drag coefficient of a flat plate with infinite cavity and zero cavitation number, the well-known result of Kirchhoff. Table I shows the variation of the different parameters and integrals.

12. Calculations of the thickness of the jet show it to be essentially constant and equal to 22% of the plate width over the considered region of cavitation numbers.

13. The length of the cavity, which is taken to be the distance between stagnation points, is plotted in units of plate width in Figure

* The preceding analysis is contained essentially in reference (b).

[illegible] the cavity [illegible]

[illegible]

[illegible] the first obstacle which prevents [illegible] formation. In fact, the total thrust on it is equal to the force on the forward obstacle, so that the total force on the finite cavity is zero, as would be expected from the D'Alembert paradox.

16. The calculation of the flow about two plates connected by a free boundary* proceeds similarly to the previous calculation. The upper half of the z-plane (i.e. the flow plane) is mapped on the w-plane as shown in Figure 4-b. Corresponding points on the two planes have the same lettering. The mapping of the [illegible] on the Q-planes is shown in Figure 4-c, which, as before is readily obtained from the conditions that the flow direction is constant on the two obstacles, while the flow velocity is constant on the free boundary. The mapping of the Q-plane on the w-plane is here achieved directly by means of the Schwarz-Christoffel transformation

$$\frac{dQ}{dw} = \frac{K_1}{\left[(w^2-a^2)(w^2-b^2)\right]^{1/2}}$$

giving

$$Q = K_1 \log \left[\frac{\dfrac{a}{\sqrt{a^2-b^2}} - \dfrac{\sqrt{w^2-b^2}+\sqrt{a^2-b^2}}{w+a}}{\dfrac{a}{\sqrt{a^2-b^2}} + \dfrac{\sqrt{w^2-b^2}+\sqrt{a^2-b^2}}{w-a}} \right]$$

The conditions

$$Q = 0 \text{ at } w = 0, \quad Q = -i\pi/2 \text{ at } w = 1$$

give $K_2 = 0$, $K_1 = -1/2$, and therefore, since $Q = \log U\frac{dz}{dw}$

* Note that the distance between the plates for a flow with preassigned cavitation number is one of the unknowns of the problems. Conversely, if the position of the plates is fixed, the cavitation number is unknown.

[illegible]

[illegible] in stating the [illegible]

$$(b)\ C_D = \text{[illegible]}$$

where now

$$(b')\ I_1 = -\int_1 (1/I)\,\text{[illegible]}$$

$$(b'')\ I_2 = \int \text{[illegible]}$$

I being given by (a) above. These integrals [illegible] in terms of elliptic integrals.

$$-i\,I_1 = \text{[illegible]}\,E(k, \pi/2) - F(k, \pi/2) \text{[illegible]} (t^2 \text{[illegible]})$$

$$i\,I_2 = t^2 E(k, \pi/2) - F(k, \pi/2) + (t^2 - 1) \qquad (k = \sqrt{t^2-1}/t)$$

where E and F are the complete elliptic integrals of the first and second kind, respectively.

17. The length of the cavity, which is the distance between stagnation points A, A' is given by

$$-\frac{i\ell}{2}\,\frac{\int^{t} I\,dw'}{\int^{t} I\,dw'}\,.$$

If the upper integral is replaced by $\int_1^t$ Idw', the real part of the resulting expression represents the half-distance between stagnation

[illegible]

[illegible] indistinguishable from one another [illegible] the exact [illegible] cavitation number is taken into [illegible].

[illegible] The evaluation of the cavity length shows [illegible] agreement between the two theories. The cavity lengths and [illegible] are tabulated in Table [illegible] in units of the plate width, and the former [illegible] plotted as a function of cavitation number in Figure 3, where [illegible] compared with the cavity length calculated from the Wagner model. The agreement between the two theories is seen here also to be remarkabl[illegible], although de[illegible] begin to appear at [illegible] cavitation [illegible] cavitation [illegible] cavity length (in units of [illegible]).

[illegible] The numerical agreement between these apparently dissimilar [illegible] is not surprising, since the only significant physical difference between the two occurs at the rear portion of the cavity, where it would not be expected to affect the drag, cavity length, or other features of the flow in the large. The finite cavities observed in water-entry or water tunnel studies differ from the two models studied [illegible] chiefly in the situation at the rear of the cavity. [illegible] of the converging streams of liquid at the rear stagna[illegible] a much weakened jet, or none at all, so that the [illegible] are intermediate in shape between the two theoretical models. Although measurements have not been made on two-dimensional cavities to confirm the results presented here, water tunnel studies on the flow about a flat disk, in which the pressures over the face of the disk were integrated to obtain the drag, show a similar linear variation of drag coefficient. This would seem to confirm the results presented here for the corresponding two-dimensional flow.

21. Mathematical solutions do not yet exist for any three-dimensional cavity flows. The applicability of analytic methods seems to be unlikely at present. The close agreement between the simplified Riabouchinski model of the cavity and the basically correct Wagner model suggests that the former will give an accurate description of the three-dimensional cavity flow, and at the same time this model offers some

hope of successful numerical treatment. A check against the adequacy of the theory can then be obtained from already available experimental results. From the mathematical point of view, it will give information [illegible] methods. [illegible]

[illegible]

D. H. ROCK

PLATE I

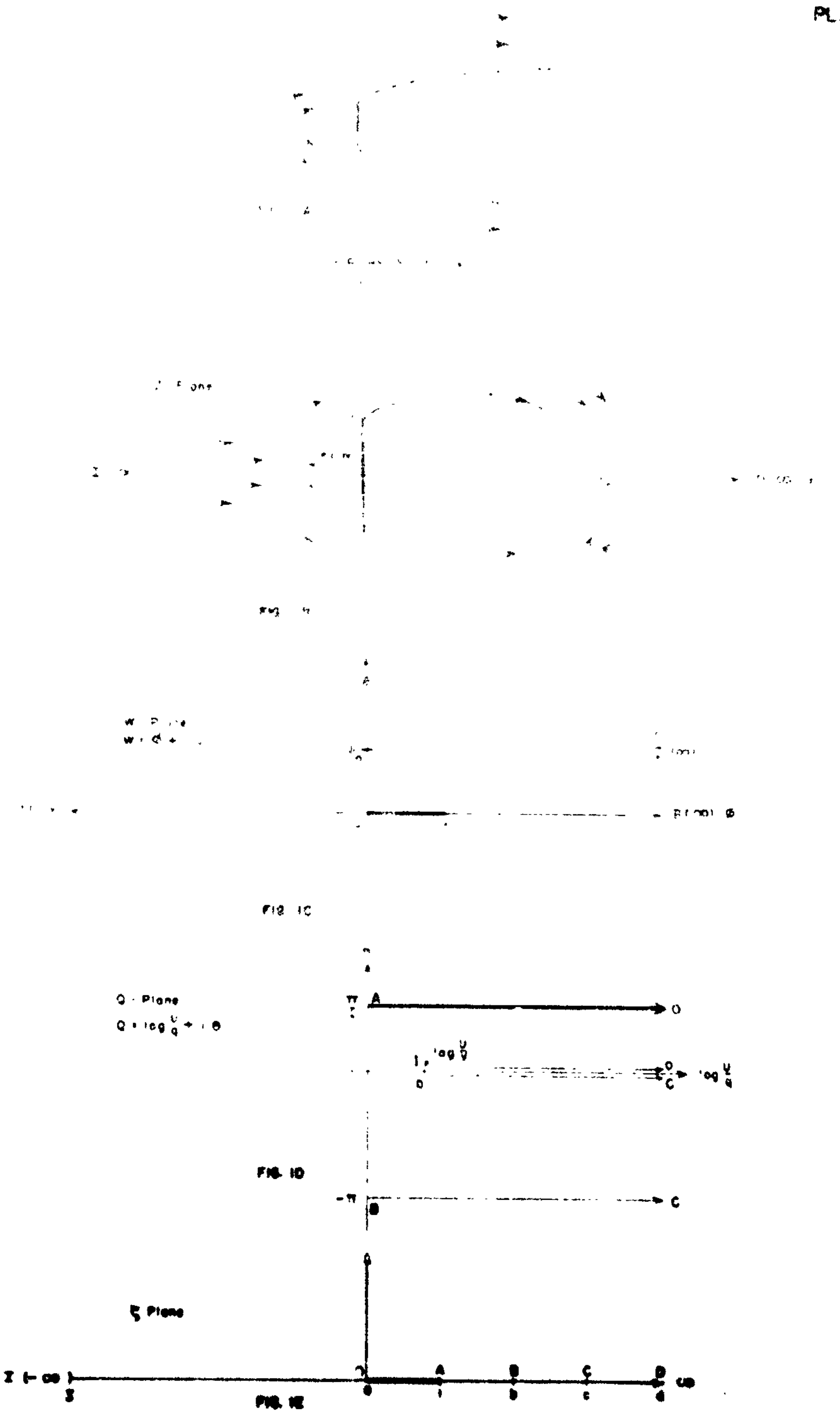
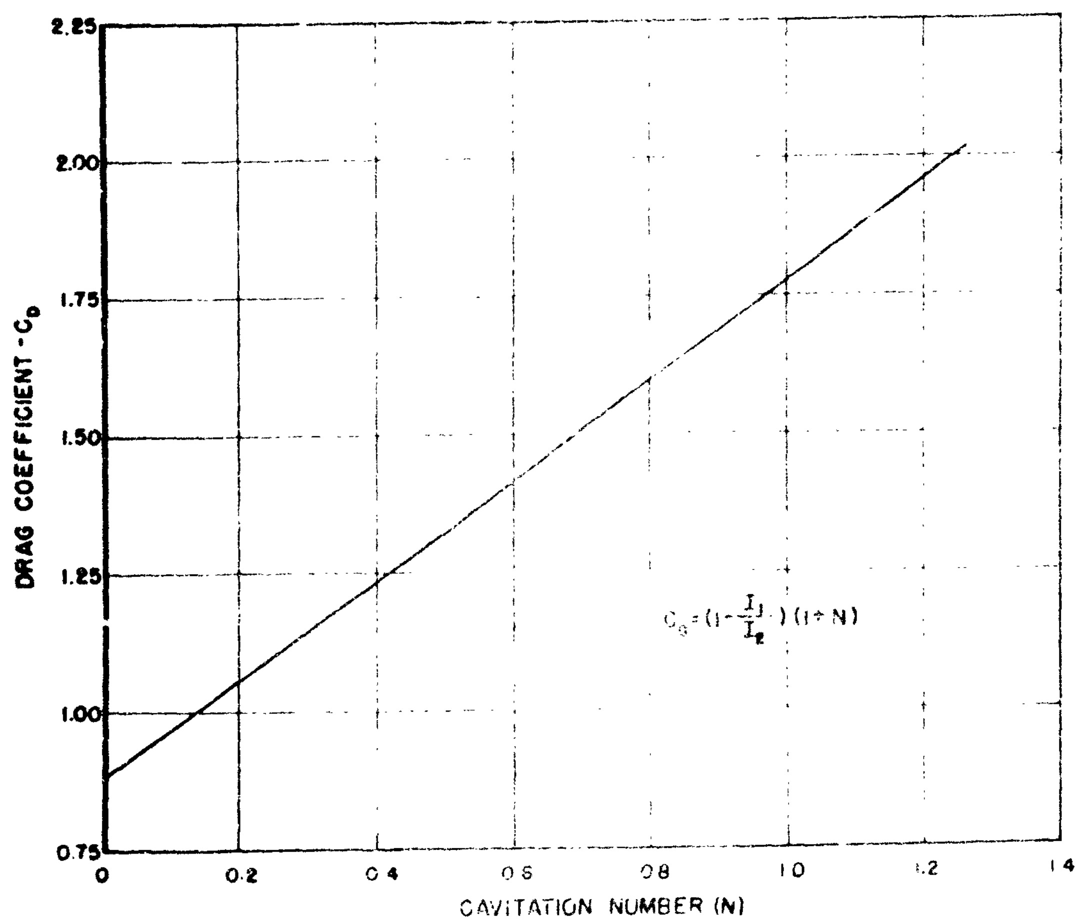


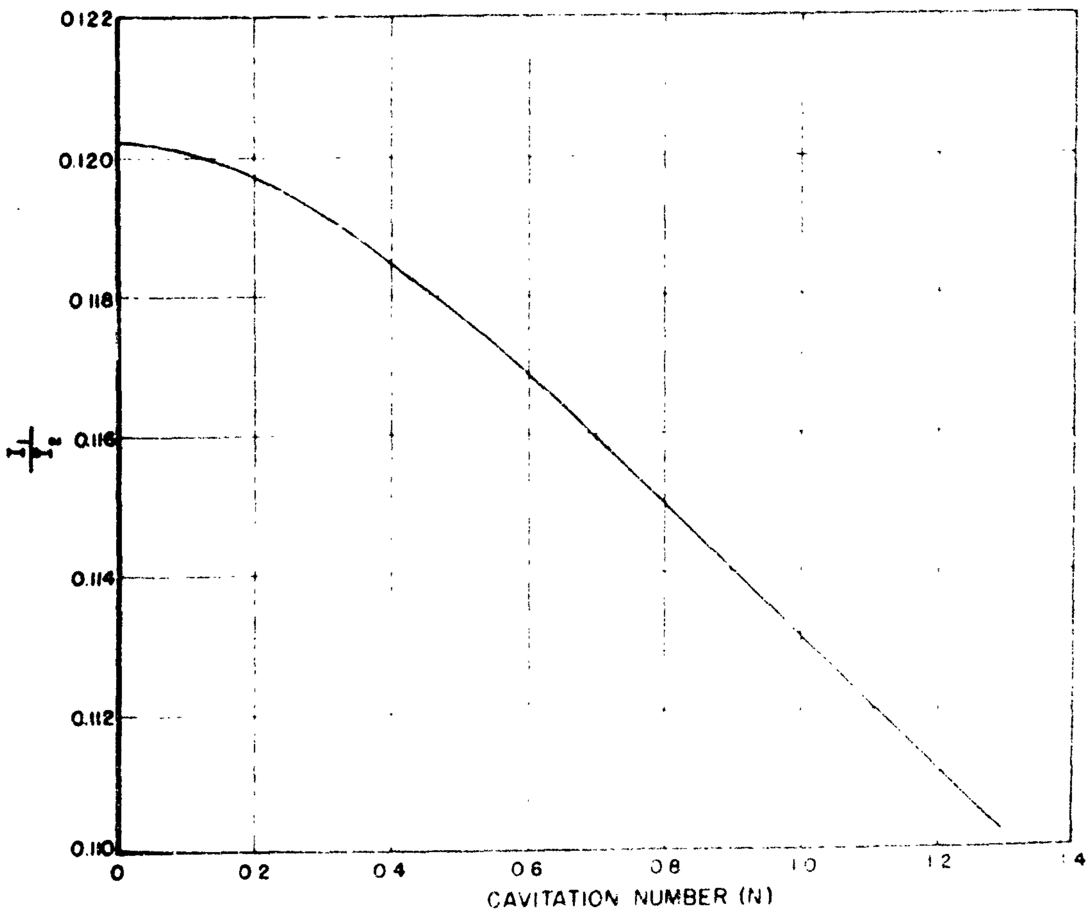
FIG. 1C

FIG. 1D

FIG. 1E



DRAG COEFFICIENT AS FUNCTION OF CAVITATION NUMBER



VARIATION OF I_1/I_2 WITH CAVITATION NUMBER

PLATE 4

FIG 4A

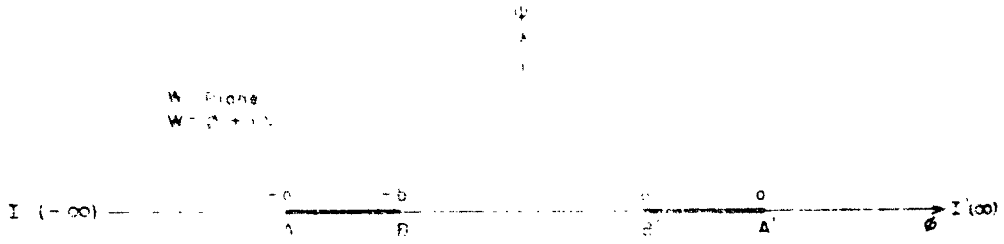


FIG 4B

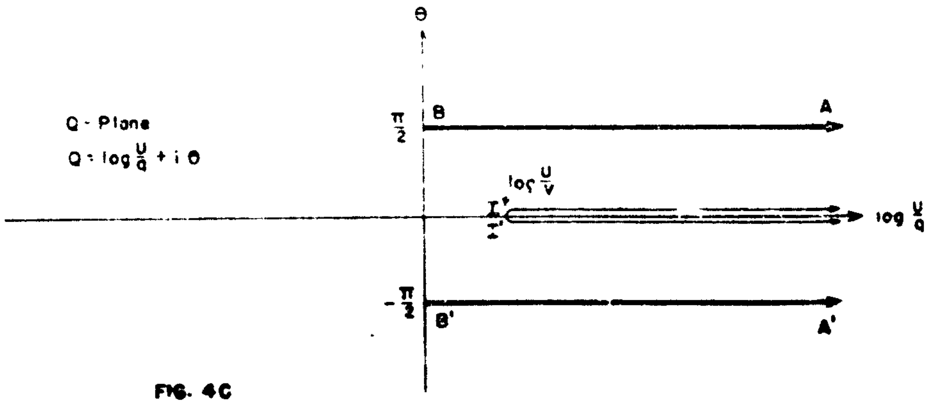


FIG. 4C

Table I

Numerical Values for [illegible] Model

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
		0.2500			[illegible]	0.22999	
[illegible]	[illegible]	0.2501	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.2504	0.4287	3.582	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	0.2508	0.4281	3.594	[illegible]	0.2192	30.97
[illegible]	[illegible]	0.2513	0.4275	3.608	0.1185	0.2188	[illegible]
[illegible]	[illegible]	0.2519	0.4267	3.624	[illegible]	0.2183	[illegible]
[illegible]	36	0.2553	0.4222	3.723	[illegible]	0.2154	[illegible]
[illegible]	[illegible]	0.2577	0.4191	3.795	[illegible]	0.2133	3.198

Table II

Numerical Values for the Riabouchinski Model

M	$II_1 \times 10^6$	$II_2 \times 10^6$	I_2/I_0	1	-1x cavity length	-1x cavity diameter
0.094	4.293	35.72	0.1202	1.001	279.1	12.97
0.196	17.14	143.2	0.1197	1.004	69.18	6.72
0.308	38.48	323.1	0.1191	1.009	30.41	4.64
0.397	59.74	504.2	0.1185	1.014	19.36	3.82
0.491	85.14	722.9	0.1178	1.020	13.41	[illegible]
0.604	118.8	1016.9	0.1168	[illegible]	9.45	[illegible]
0.695	148.1	1276.4	0.1160	[illegible]	7.48	[illegible]
0.794	181.3	1575.6	0.1151	1.043	6.02	2.40
0.901	218.5	1915.6	0.1141	1.052	4.92	2.24
1.004	255.4	2259.0	0.1131	1.061	4.15	2.11